

# THE PRIMARY MATHEMATICS RESEARCH PROJECT

## TOWARDS EVIDENCE-BASED EDUCATIONAL DEVELOPMENT IN SOUTH AFRICA

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*Educating for impact in mathematics, science and language*



Eric Schollar and Associates

## **Motivation for the PMRP**

The persistence of poor outcomes in mathematics education in South African schools despite the post-1994 dispensation:

❖ the introduction of the new Outcomes Based Education curriculum in 1998

❖ the ever-increasing allocation of significant national resources to the education system

## **Learner Performance**

National and international data has demonstrated that the vast majority of South African learners are performing well below the minimum expected competence levels for their respective grades.

This reality ultimately manifests itself at higher grade matriculation level - only 1.5% of the 1995 Grade One cohort survived to achieve HG pass in the 2006 matriculation examinations.

# Learner Performance

Learners at different achievement levels of the  
NCS: National Systemic Grade 6 Cycle (%)

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	Test score range	Mathematics
Not achieved	1% to 39%	81
Partly achieved	40% to 49%	7
Achieved	50% to 69%	8
Outstanding	70% to 100%	4

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## Learner Performance

When similar figures are obtained in both national and international studies, powerful support is provided to all of them.

% of learners below minimum competence level

NSE	SACMEQ	TIMMS
Grade 6: 2005	Grade 6: 2000	Grade 8: 2003
81	84	82

The SACMEQ study found that 52% of Grade 6 learners were achieving scores in mathematics at the Grade 3 level or lower.

# Learner Performance: Conclusions

- ❖ Assessment policies and practices have failed to produce a reasonable degree of ‘fit’ between the expected and actual performance levels of learners.
- ❖ Learners are routinely promoted from one grade to the next without having mastered the content and foundational competences of preceding grades.
- ❖ Every class has become, in effect, a ‘multi-grade’ class in which there is a very large range of learner abilities making it very difficult, or even impossible, to consistently teach to the required assessment standards for any particular grade.

# International Context

Poor learner performance in mathematics is not unique to South Africa as TIMSS has made clear. Alarm has been raised in many countries, the United States and Australia being the most recent examples.

The recent research and policy literature flowing from these countries has an increasing focus on the nature of the curriculum, the learning theory upon which it is based and the teaching practices that it encourages.

Constructivism is under increasing pressure to provide reliable empirical evidence that it is an effective theoretical basis for a national curriculum, especially for the teaching of the fundamentals of mathematics to young learners in primary schools.

## South African Context

Recent reviews of evaluation studies of various interventions have shown that the majority of these studies agree that:

- ❖ significant changes in teacher and learner behaviour were achieved, these changes were both intended and promoted by the interventions and they were consonant with constructivist practices
- ❖ very few of the interventions achieve equally consistent impact upon learner performance



## South African Context

Recent research, including production function analysis, indicates that there are a number of causative factors including:

- ❖ incomplete coverage of the curriculum
- ❖ the ‘localization’ of assessment
- ❖ the ‘localization’ of the syllabus of content
- ❖ poor teacher content knowledge combined with the virtual abolition of textbooks

## South African Context

- ❖ insufficient opportunities for regular and extensive practice of content by learners, especially in terms of reading, writing and solving mathematical problems
- ❖ insufficient level of monitoring by local-level DoE of management of curriculum and assessment by schools - emphasis is on compliance with formal policy & production of documents rather than quality

# PMRP: Phase I: Objective and Data Source

To carry out an empirical investigation into the nature of the outcomes of mathematics education in primary schools.

Data sources consisted of:

- ❖ the individual item scores obtained from 7 028 learners from 154 schools in 24 districts in all 9 provinces between 1998 and 2004
- ❖ the original rough workings used by 4 256 of the learners between 2002 & 2004

Scripts were drawn from evaluations of 6 different studies of intervention projects, using the same test instrument. The learner datasets were supported by the data sets of interviews and, especially, lesson observations, conducted during the same studies.

# PMRP Phase I: Findings

Phase I confirmed the poor levels of learner performance measured in other studies, especially in Learning Outcome One (see National Systemic).

The analysis of rough workings distinguished between three methods used in the solving of problems:

❖ *Unit counting*: all kinds of problems are solved by reducing the numbers involved to single unit marks and counting them one-by-one

❖ *Repeated Operations*: multiplication & division problems are solved using whole numbers, but are reduced to addition and subtraction by repeatedly adding or subtracting the numbers involved

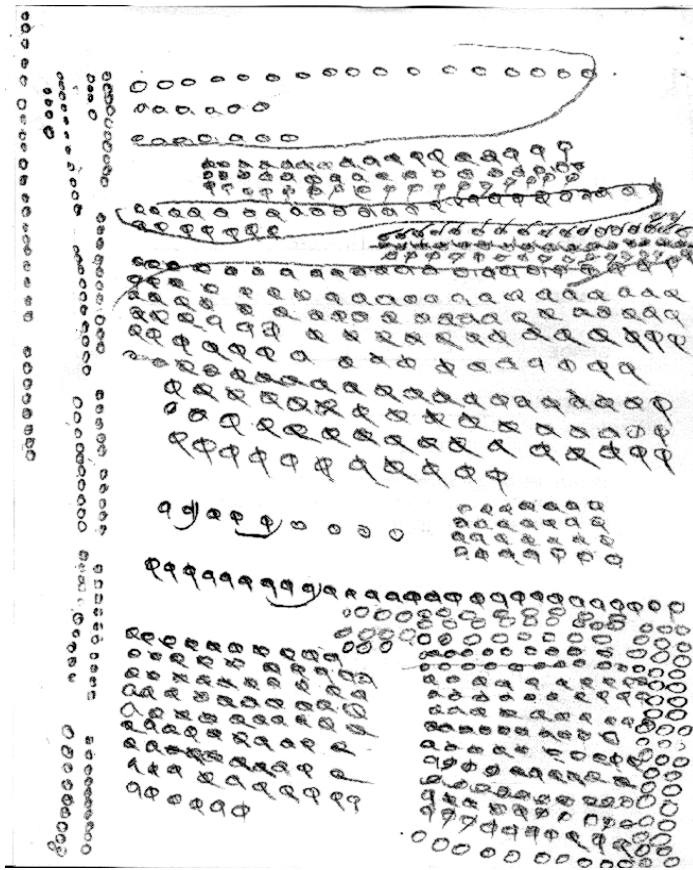
❖ *Calculations*: all kinds of problems are solved using whole numbers to calculate - as against count - the solutions.

## PMRP Phase I: Findings

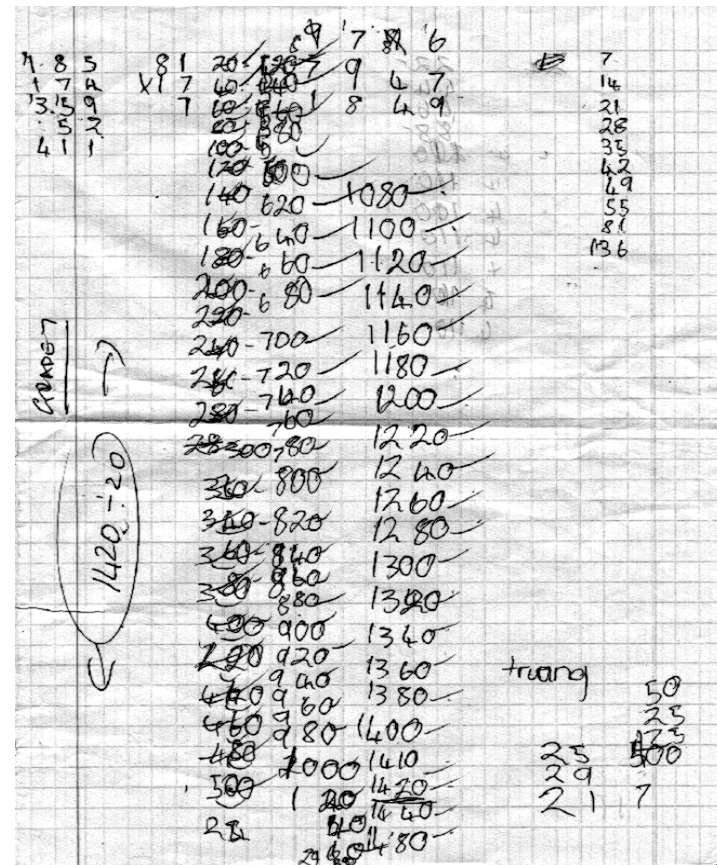
- ❖ 79.5% of Grade Five and 60.3% of Grade Seven children still relied on simple unit counting to solve problems to one degree or another
- ❖ 38.1% of Grade Five and 11.5% of Grade Seven children relied *exclusively* upon this method.

# Phase I: Examples of Problem Solving

## Unit counting



## Repeated operations



# Phase I: Examples of Problem Solving

Inability to calculate

	T	U
3		6
3		4
3		1

## Phase I: Lesson Observation

*‘If you are asked 15 times four its meaning is that you are counting 15 four times. To do this you must first expand the sum.’*

Writes on board:  $15 \times 4 = (10 \times 4) + (5 \times 4) = (10 + 10 + 10 + 10) + (5 + 5 + 5 + 5) = (1111111111 + 1111111111 \text{ –four times}) + (11111 + 11111 \text{ –four times})$  Counts units =  $(40) + (20)$  ‘When you add 40 and 20 what do you get?’ First group to answer 60 is asked to show workings on board: pupil comes to board and writes  $(1111111111 \text{ – four times}) + (1111111111 \text{ – twice})$ . Counts individual units and writes = 60

Each group is given a sum. Workings of first group to report:  $23 \times 7 = (20 \times 7) + (3 \times 7) = (1111111111111111111111111111 + 1111111111111111111111111111 \text{ –seven times}) + (111 + 111 \text{ –seven times})$ . Counts units = 160 [correct answer is 161]



# Phase I: Conclusions

The fundamental causes of poor performance are:

- ❖ a failure to extend the ability of learners from counting to true calculating. All more complex mathematics depends on an understanding of place value within the base-10 number system, the ability to readily perform basic calculations and see numeric relationships.
- ❖ this problem is caused by the application of ineffective learning practices in classrooms resulting in the virtual disappearance of memorization, consistent drill and regular extensive practice of learned content
- ❖ learners are not being given the opportunity to develop the neural pathways and structures required for the development of higher order cognitive competencies in mathematics

## **Phase I: Conclusions**

Closely associated with these causes has been the virtual abolition of the concept of a national or provincial syllabus of study combined with textbooks designed to give effect to this syllabus.

Quality of outcome has varied wildly from school to school as the completeness and complexity of content to which learners are exposed came to depend on individual teachers.

The vast majority of our classes have become, in effect, multi grade classes in which teachers are faced with learners with every conceivable level of ability from the innumerate to the genuinely competent.

## Phase II of the PMRP: Materials

The development of a set of teacher and learner materials based on the findings of Phase I.

These materials are based on a number of key design features, they:

- ❖ are concerned with Learning Outcome One only – Numbers, Operations and Relationships

- ❖ are based on experimentation with an approach emphasizing direct instruction by teachers, combined with the use of memorization, drill and extensive regular applied practice for learners - before extensions into more complex 'learner-centred' activities (like games and puzzles, etc.) are attempted

## **Phase II of the PMRP: Materials**

- ❖ provide a grade-differentiated capacity allowing for teaching in classrooms where learners have widely differing levels of subject competence
- ❖ provide a diagnostic and formative assessment system to control the exposure of learners to the correct complexity level in practice of learned content
- ❖ provide teachers and learners with a complete syllabus of study, backed by a complete set of materials, based on the Assessment Standards of the NCS

## **Phase II: Field Testing of Materials**

Experimental design collecting both quantitative and qualitative data

40 schools from 3 circuits in Limpopo. Random selection of 20 project and 20 control schools

Two classes, one each from Grade 4 & 6 from each school

Total n numbers: 3 032 learners with 1 560 in the project group & 1 472 learners in the control group

## Phase II: Test Instruments

The test instruments (Grade 4: 56 items) Grade 6: 86 items) were constructed from a number of different sources:

- ❖ the items dealing with LO One from the previous version of the National Systemic Evaluation
- ❖ the LO One items from the instruments regularly used by ESA, and upon which Phase I was based
- ❖ the development of 8 simple word sums matched to 8 operations
- ❖ for the post-tests, 20 items dealing with the four operations were developed in the Grade 6 instrument to measure the degree of difference between groups in terms of items based on the assessment standards for Grades 5 and 4.

# Phase II: Indicators of Impact on Learner Performance

- ❖ A statistically significant (i.e. over +2% in % points) increase in score of the project group over the control group between pre-and post-testing
- ❖ A significant difference in the frequency of calculation methods, as against counting methods, in the project over the control group by the end of the programme
- ❖ Significant impact measured in over 80% of schools & for more than 80% of learners

## Phase II: Exposure to Intervention

Since the materials are based on a sequential and cumulative approach to the teaching and learning of mathematical content, it was essential to know the level of exposure of learners to the full ‘treatment’ – i.e. coverage of the ‘curriculum’.

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	Grade 4	Grade 6
Whole sample (schools)	20	20
Minimum of 7 weeks coverage (% of schools)	80	85
Minimum of 11 weeks coverage (% of schools)	60	50

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## **Phase II: Findings**

The study provided strong and reliable empirical evidence that the theoretical and methodological approach embodied in the PMRP materials results in rapid and significant improvements in learner performance in Learning Outcome One. These improvements were obtained over a period of 14 weeks.

The report presents a mass of impact data that can be summarized in the greater degree of increase of score over baseline of the project group when compared to the control group - whether measured in terms of % point increase or % increase over baseline.

## PHASE II: Findings: Mean Scores: Summary

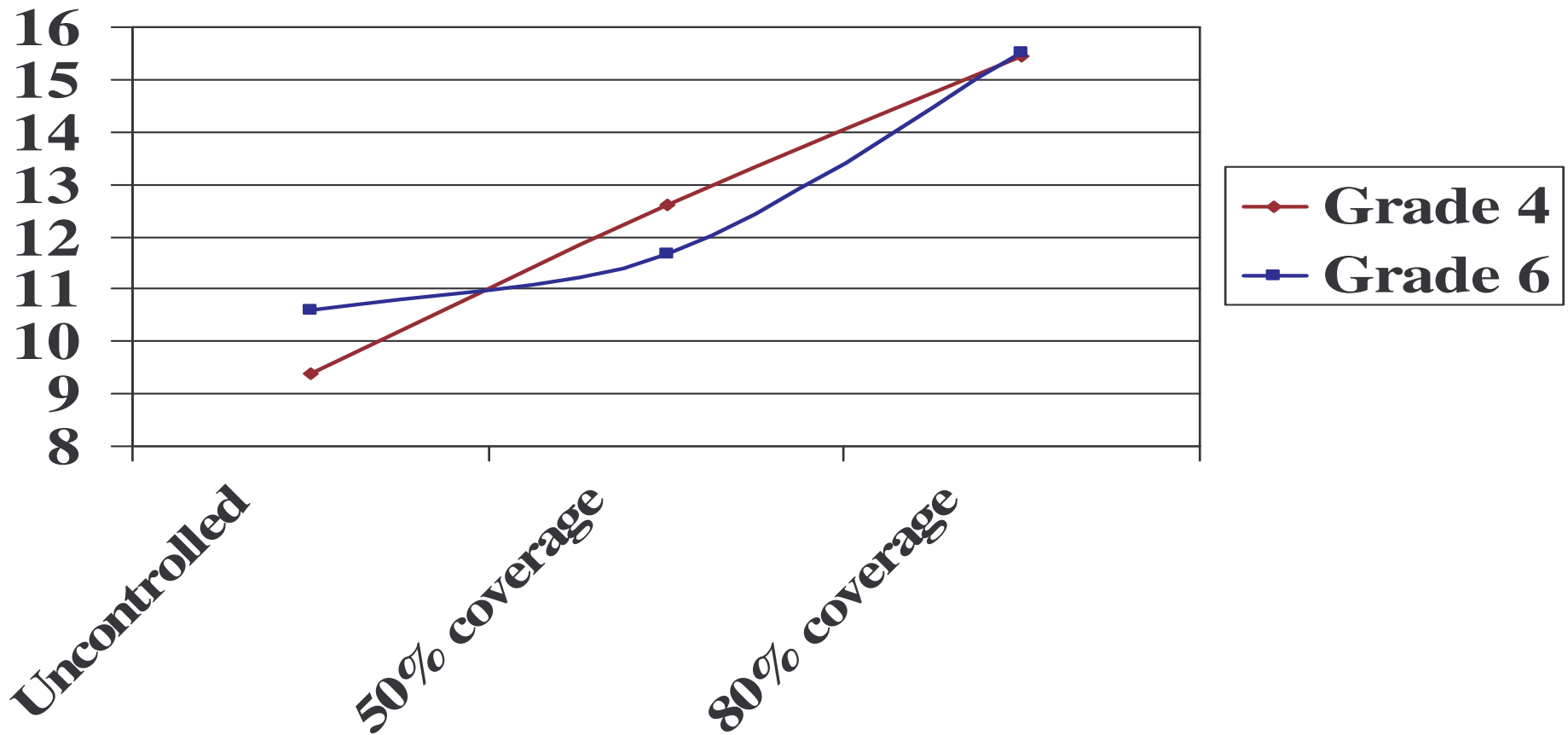
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	Grade 4	Grade 6	Combined
<hr/>			
Uncontrolled for coverage			
Impact: % points	+9.39	+10.59	+9.99
Impact: % over baseline	+49.90	+63.77	+56.83
<hr/>			
Controlled: 50% of coverage			
Impact: % points	+12.59	+11.68	+12.13
Impact: % over baseline	+72.01	+66.83	+69.42
<hr/>			
Controlled: 80% of coverage			
Impact: % points	+15.44	+15.60	+15.52
Impact: % over baseline	+82.74	+101.76	+92.25

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# PHASE II: Findings: Mean Scores: Summary

Increases are significant for all levels of exposure but the generic effect of increased curriculum coverage is very clear. (The graphic uses % point increase)



## Learner Performance on NSE Items: Controlled = 80% (11 weeks) of coverage from here on

	Grade 4 (12 items)		Grade 6 (16 items)	
	Uncontrolled	Controlled	Uncontrolled	Controlled
<b>Project</b>				
Baseline	23.17	21.58	20.19	18.50
Post Test	30.00	36.83	25.38	32.12
<i>Change</i>	+6.83	+15.25	+5.19	+13.62
<i>% increase on baseline</i>	+29.50	+70.67	+25.70	+73.62
<b>Control</b>				
Baseline	23.42	23.42	17.75	17.75
Post Test	17.92	17.92	19.75	19.75
<i>Change</i>	-5.50	-5.50	+2.0	+2.0
<i>% increase on baseline</i>	-23.48	-23.48	+11.27	+11.27
<b>Impact: % points</b>	<b>+12.33</b>	<b>+20.75</b>	<b>+3.19</b>	<b>+11.62</b>
<b>Impact: % increase</b>	<b>+52.98</b>	<b>+94.15</b>	<b>+14.43</b>	<b>+62.35</b>

# Learner performance in the Four Operations

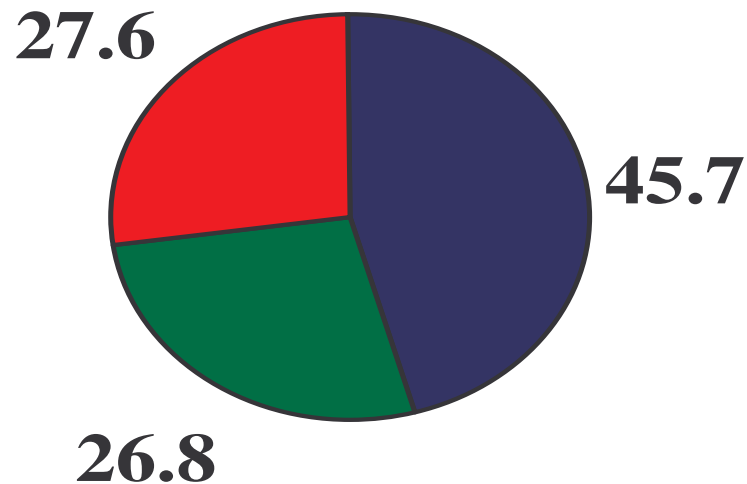
	Grade 4		Grade 6	
	Uncontrolled	Controlled	Uncontrolled	Controlled
<b>Add</b>				
% points	+7.2	+9.4	+18.2	+23.4
% of baseline	+16.43	+20.0	+42.18	+54.86
<b>Subtract</b>				
% points	+9.00	+11.2	+17.4	+25.0
% of baseline	+32.90	+34.66	+87.91	+121.32
<b>Multiply</b>				
% points	+9.0	+13.0	+11.8	+18.6
% of baseline	+117.09	+184.12	+378.57	+814.29
<b>Divide</b>				
% points	+12.6	+16.4	+10.2	+13.8
% of baseline	+92.24	+104.94	+128.15	+156.01

# Grade 6: Performance against Assessment Standards for Grades 4 and 5: Post Test Only

	<b>Uncontrolled</b>	<b>Controlled</b>
Project	38.5	45.1
Control	17.5	17.5
<b>Difference</b>	<b>+21.0</b>	<b>+27.6</b>

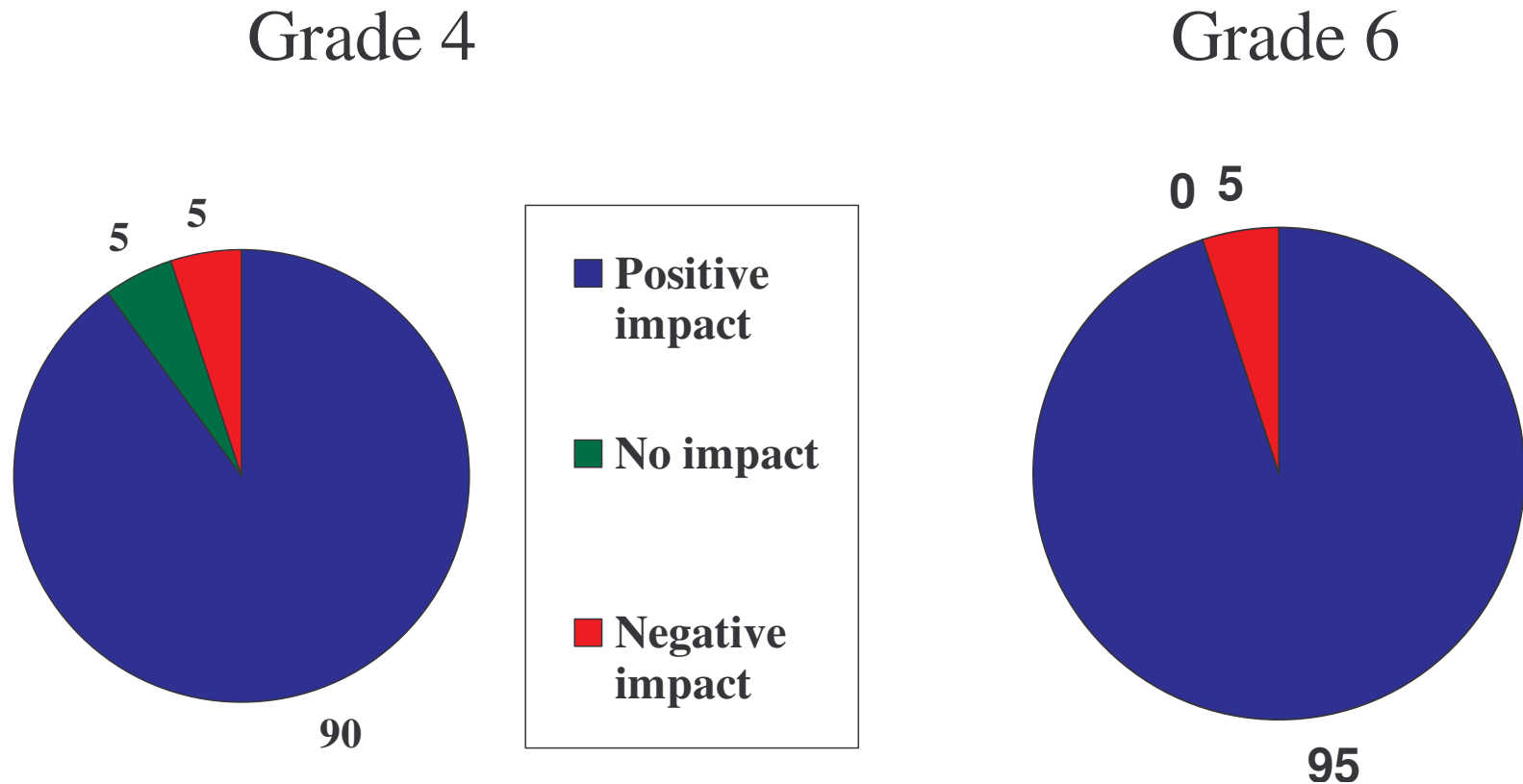
## Distribution of Impact: Schools

Nick Taylor has shown that significant impact on learner performance is typically recorded in only a fairly small proportion of schools – around 50%. The graphic is based on a recent 4-year project.



# PMRP: Distribution of Impact: Schools

Both graphics are based on data uncontrolled for coverage



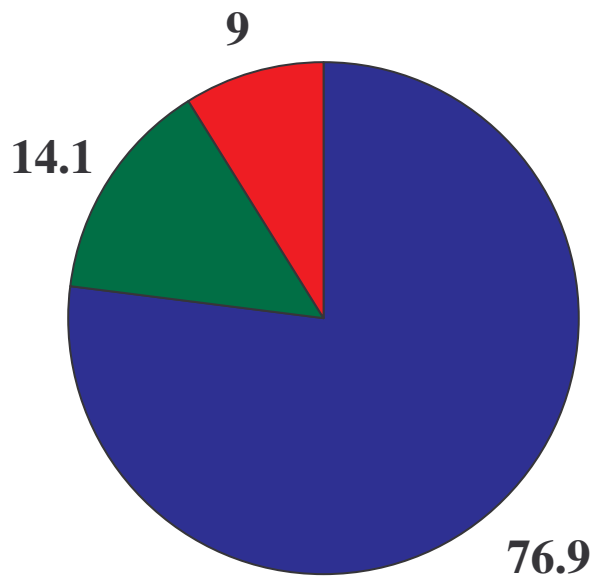
Once the data are controlled for coverage, all 100% of the schools recorded significant impact



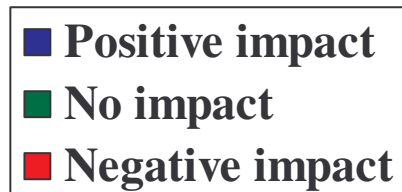
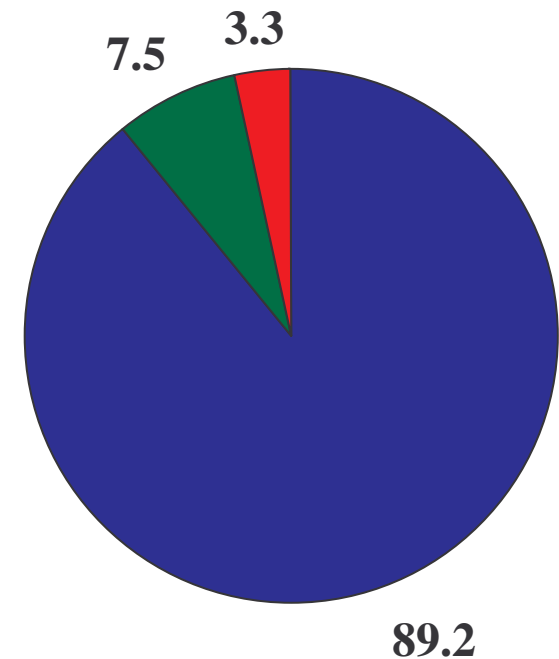
# PMRP: Distribution of Impact: Individual Learners

In these graphics, scores are drawn from the combined group of Grade 4 and 6 learners.

Uncontrolled for coverage

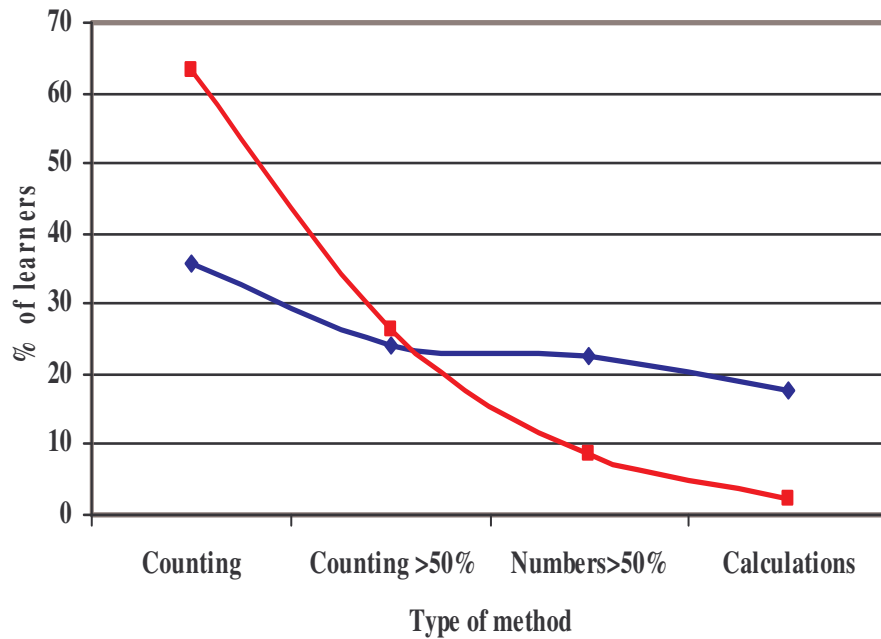


Controlled for coverage

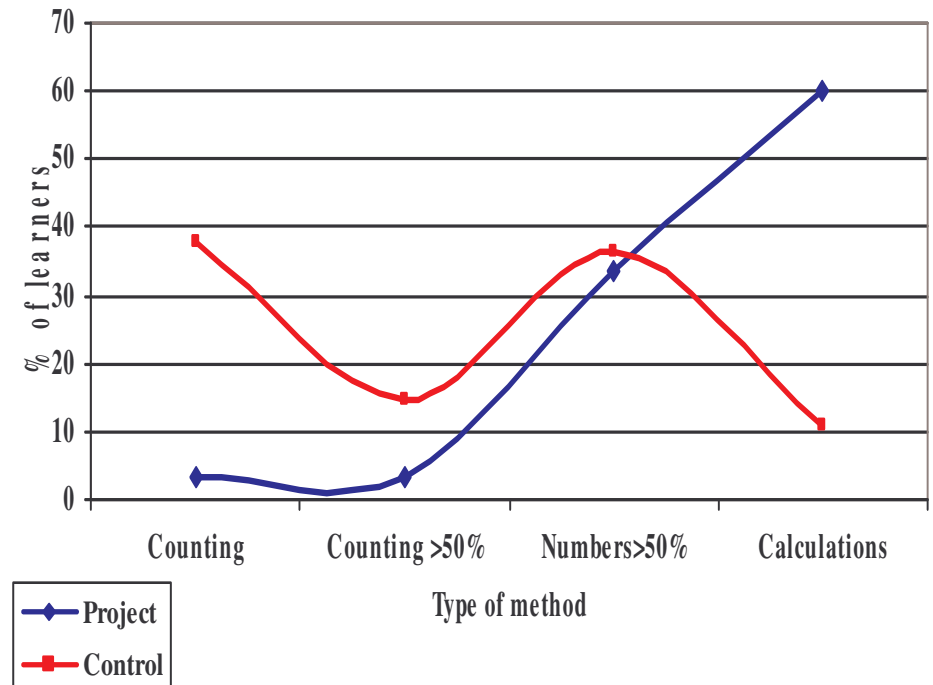


# Frequency of Problem-Solving Methods: % of Learners

## Grade 4

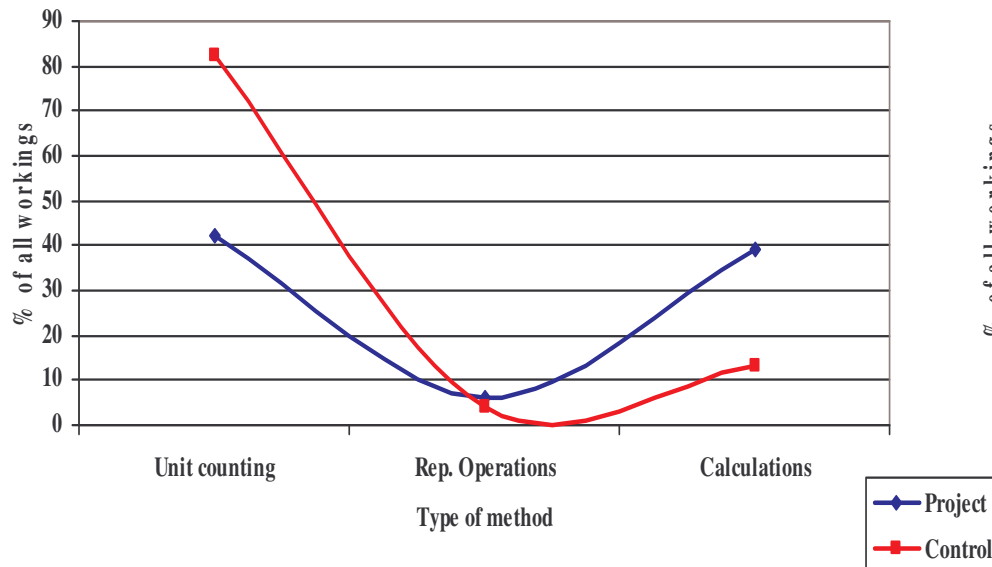


## Grade 6

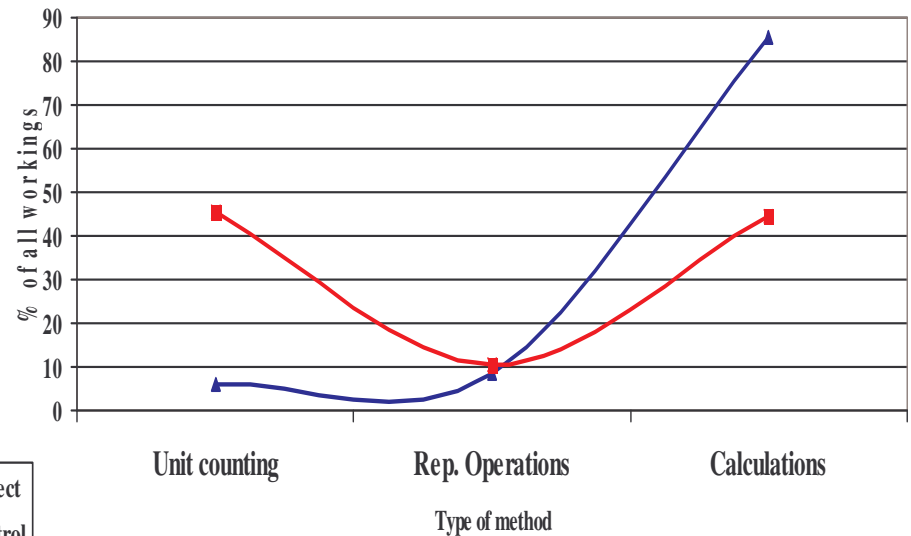


# Frequency of Problem-Solving Methods: % of all Attempts

## Grade 4



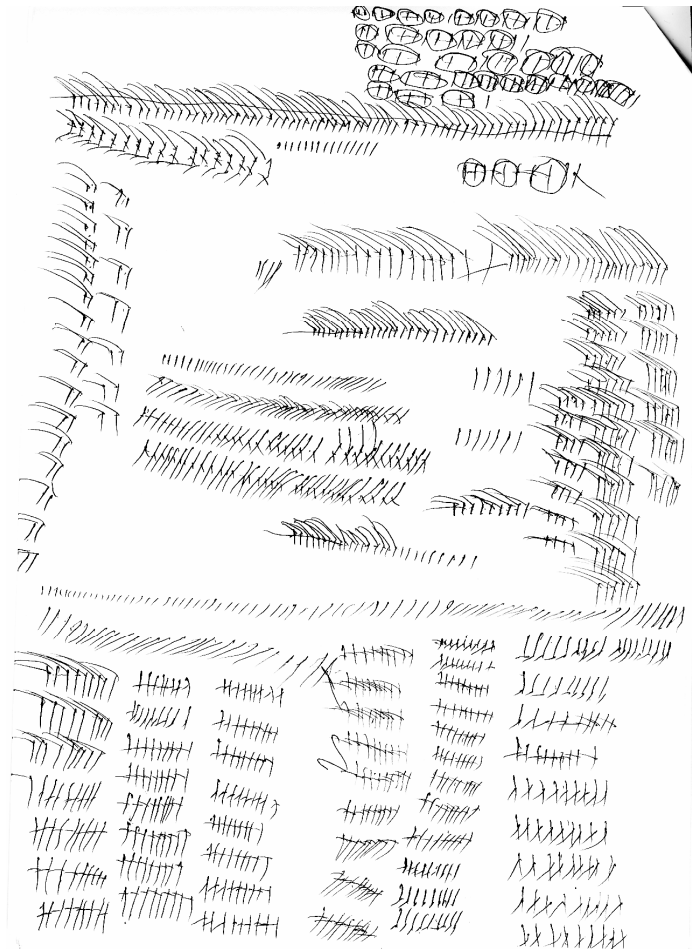
## Grade 6



# Problem-Solving: Scans from Learner Scripts

These examples are both drawn from Grade 6 scripts.

Control



Project

$$\begin{array}{r}
 1 \\
 75 \\
 + 36 \\
 \hline
 111
 \end{array}$$

$$\begin{array}{r}
 2^3 \\
 256 \\
 \times 15 \\
 \hline
 1280 \\
 2560 \\
 \hline
 3840
 \end{array}$$

$$\begin{array}{r}
 11 \\
 527 \\
 + 394 \\
 \hline
 921
 \end{array}$$

$$\begin{array}{r}
 4 \overline{) 12} \\
 3 \overline{) 123} \\
 1 \\
 23 \\
 \hline
 115 \\
 230 \\
 \hline
 345
 \end{array}$$

$$\begin{array}{r}
 11 \\
 3795 \\
 + 4574 \\
 \hline
 8369
 \end{array}$$

$$\begin{array}{r}
 13 \\
 \times 13 \\
 \hline
 39 \\
 130 \\
 \hline
 169
 \end{array}$$

$$\begin{array}{r}
 2216 \\
 + 2681 \\
 \hline
 4897
 \end{array}$$

$$\begin{array}{r}
 85 \\
 482 \\
 - 799 \\
 \hline
 163
 \end{array}$$

$$\begin{array}{r}
 8 \\
 45 \\
 - 78 \\
 \hline
 17
 \end{array}$$

$$\begin{array}{r}
 86 \\
 6976 \\
 - 3898 \\
 \hline
 3078
 \end{array}$$

$$\begin{array}{r}
 758 \\
 - 527 \\
 \hline
 231
 \end{array}$$

# Problem-Solving: Scans from Learner Scripts

The scans on the left typify methods used in control schools: 4x7 and 5x67. The scan on the right is from a project school: 856x45 and 8 681x37.

4 4 4 4 4 4 4  
 1111 1111 1111 1111 1111 1111 1111

~~55555555~~  
~~55555555~~ 555555555555  
~~55555555~~ 555555555555  
~~55555555~~ 555555555555

856 x 45	= 38520 ✓
8 681 x 37	= 321197 ✓

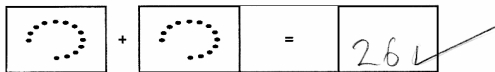
$$\begin{array}{r} 856 \\ \times 45 \\ \hline 4280 \\ 34240 \\ \hline 38520 \end{array}$$

$$\begin{array}{r} 8681 \\ \times 37 \\ \hline 160767 \\ 260430 \\ \hline 321197 \end{array}$$

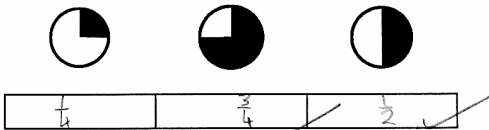
# Problem-Solving: Scans from Learner Scripts

Learners from project schools are able to correctly answer questions from Part 2 (ESA) and Part 1 (NSE), respectively

15. How many dots can you see? Write your answer in the empty box.



16. What fraction of each circle has been coloured black? Write your answers in the empty boxes below each picture.



17. Round off the following numbers to the nearest 10. The first question has been done for you.

Example	109	rounds off to	110
	6	rounds off to	10
	57	rounds off to	60
	43	rounds off to	40

18. Complete the following sequences. The first question has been done for you.

Example	1	2	3	4	5	6
	2	4	6	8	10	12
	15	12	9	6	3	0
	150	170	190	210	230	250
	225	200	175	150	125	100

Total: Part Three  
11  
Learners do not write in this box

8. Write the following number in expanded notation:

$$98\ 547 = 90\ 000 + 8\ 000 + 500 + 40 + 7$$

9. Look at the following table of Roman numbers:

L	C	D	M
50	100	500	1000

What does MDCIX represent in our number system?

- A. 1 509
- B. 2 609
- C. 2 611
- D. 2 710

10. Consider the figure below:



Express the ratio of the shaded to non-shaded areas as a fraction:

$$\frac{3}{5}$$

11. Complete the following:

$$315 + \boxed{440} > 437$$

12. Write the following number in words:

12 006

Twelve thousand and six

13. Say whether the following numbers form a sequence:

8; 4; 2; 1;  $\frac{1}{2}$ ;  $\frac{1}{4}$ ;  $\frac{1}{8}$ .

- A. Yes
- B. No

# Assessment: Scans from Learner Scripts

An illustration of the enormous differences in ability in Grade 6 learners; both were passed as competent at the end of Grade 5.

**THE PRIMARY MATHEMATICS RESEARCH PROJECT**

**GRADE 6**

SCHOOL NAME TRINITY TRUST

LEARNER NAME MPHO

Learners should not write anything below this line

Group  Project  Control

School ID Number

Learner ID Number

	Part One	Part Two	Part Three	Part Four	Part Five	Total
Score	0	0	0	0	0	0

Score: Matched Items: Part Two

Scores: Part Two

Question 17 (Add)	Question 18 (Subtract)	Question 19 (Multiply)	Question 20 (Divide)
0	0	0	0

Eric Schollar and Associates

Items in Part One are from the previous version of the National Systemic Evaluation Grade 6 instrument and are used with the kind permission of the National Department of Education.

**THE PRIMARY MATHEMATICS RESEARCH PROJECT**

**GRADE 6**

SCHOOL NAME HISCKELANI PRIMARY SCHOOL

LEARNER NAME HUNGWANI DONDZI

Learners should not write anything below this line

Group  Project  Control

School ID Number

Learner ID Number

	Part One	Part Two	Part Three	Part Four	Part Five	Total
Score	8	20	20	8	167	73

Score: Matched Items: Part Two

Scores: Part Two

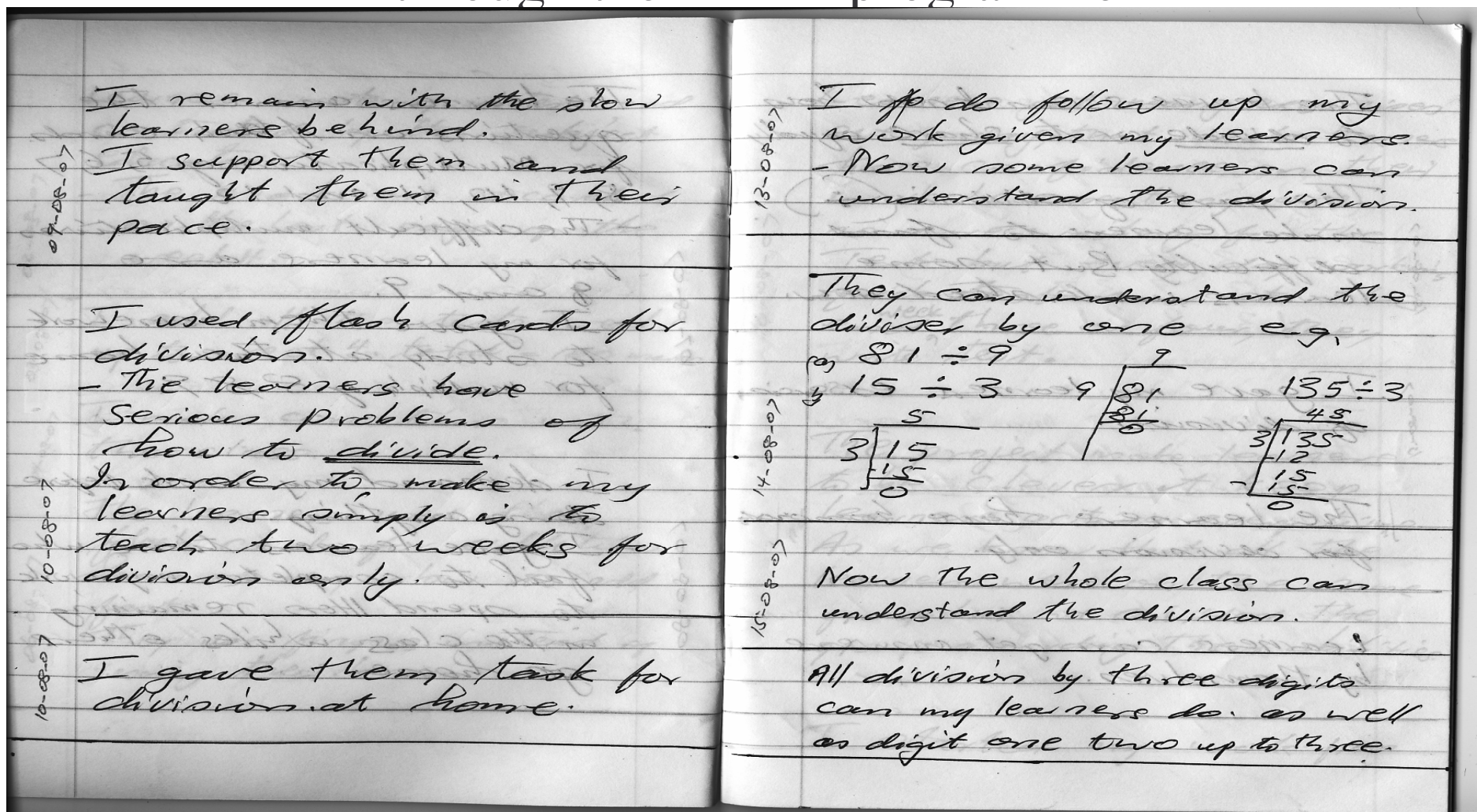
Question 17 (Add)	Question 18 (Subtract)	Question 19 (Multiply)	Question 20 (Divide)
5	5	5	5

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Items in Part One are from the previous version of the National Systemic Evaluation Grade 6 instrument and are used with the kind permission of the National Department of Education.

# Scans from Teacher research Diaries

Teachers were asked to keep a research diary as they worked through the PMRP programme





# Scans from Teacher research Diaries

This practice helped convince teachers they were part of the research team and provided useful formative information

24.05-2007

Finding from diagnostic test.

Grade 4

- Learners were too slow to finish the test. They took more than two hours to complete the whole work. Most of the learners were using small sticks to do the calculations. The highest mark in this test was 26 and lowest mark was 0. Multiplication table is the problem in this class.

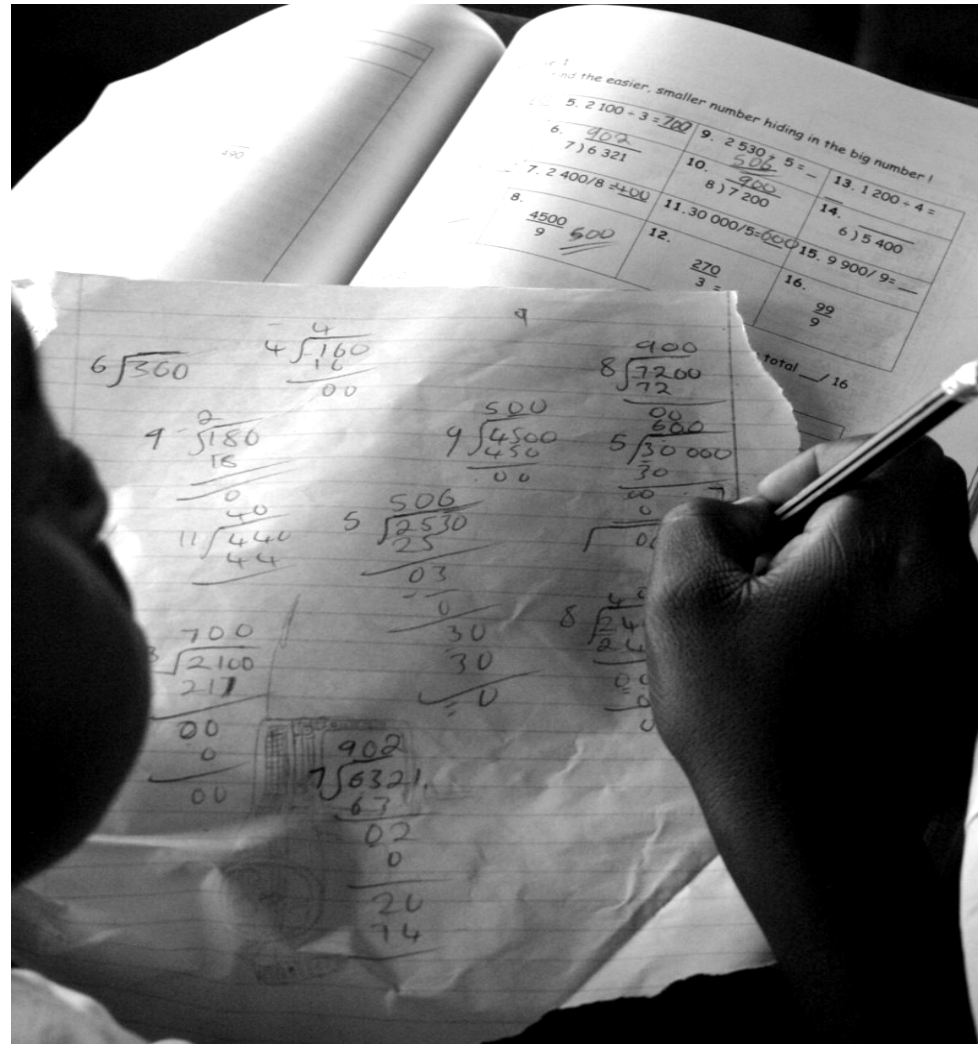
Grade 6

Grade 6 learners like grade 4 learners were also too slow. They also took two hours to complete the whole work.

- 10-09-07
- The project make learners be happy
  - The project make the educator simple to do his work.
  - No time wasted because he/she cannot write on the chalkboard because all the exercises are in their learners hand books and the space of answers are there. He can write one or two examples on the chalkboard and the rest are there in the learner's books
  - These project encourage all learners from the slow learner to the fast learners because at the end of the

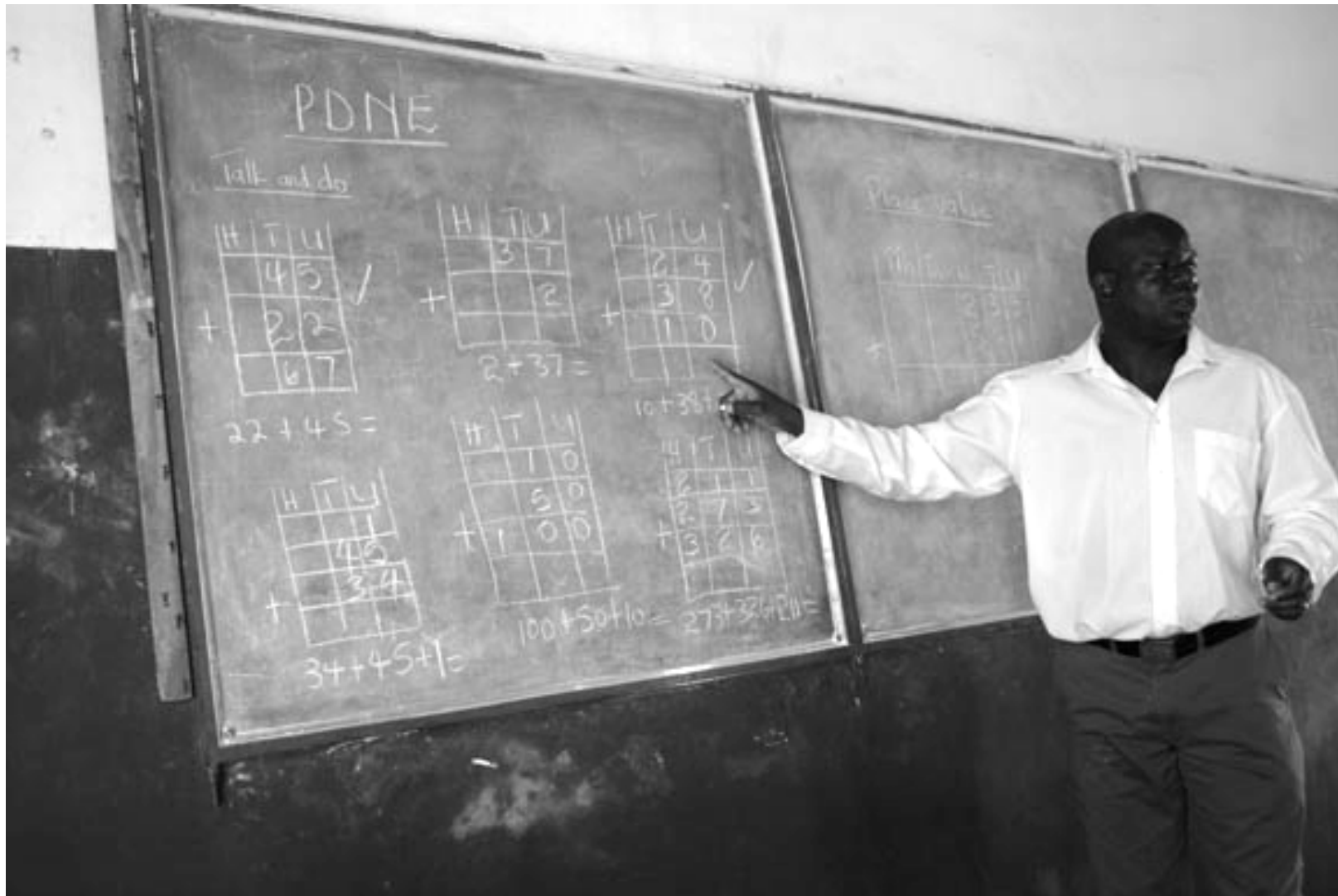
# Photographs from the Field

A learner practices the solving of many division problems in one lesson using whole number calculations



# Photographs from the Field

Direct instruction: the teacher explicates place value



# Photographs from the Field

Group work: winner of mental arithmetic game



Working as individuals



# Photographs from the Field

Working in pairs

